

$TM_{0Y\delta}$  MODE OF CYLINDRICAL DIELECTRIC RESONATORS  
APPLICATIONS TO MICROWAVE FILTERS

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### ABSTRACT

The aim of this paper is to describe an analytical method which permits to characterize the  $TM_{0Y\delta}$  mode (frequency, quality factor) and to show that we can use the dielectric resonators, excited in this mode, in many applications.

### INTRODUCTION

The dielectric resonators of cylindrical shape excited in the  $TE_{0Y\delta}$  mode were used in most of microwave applications while the  $TM_{0Y\delta}$  mode was forsaken. The present paper describes a simple analytical method which permits to characterize the  $TM_{0Y\delta}$  mode and presents results of different couplings possible with a dielectric resonator (coupling between two dielectric resonators and quality factor of a waveguide below cut-off loaded by a dielectric resonator excited by a propagating waveguide).

### DETERMINATION OF RESONANT FREQUENCY AND Q-FACTOR OF A DIELECTRIC RESONATOR EXCITED IN $TM_{0Y\delta}$ LOCA- TED INSIDE A MICROSTRIP STRUCTURE

#### 1. Resonant frequency

For the cylindrical dielectric resonators, the boundary conditions cannot be expressed by a sim-

ple analytical method ; so we must used approximations. In this analytical approach, we suppose that all the walls of the resonator are not perfectly walls ( $\vec{n} \cdot \vec{E} = 0$  ;  $\vec{n} \wedge \vec{H} = 0$ ), thus there are evanescent fields outside the dielectric object. The figure (1) presents the structure.

With the help of Helmholtz' and Maxwell' equations, we can find all the fields of the different regions.

When all the fields are known, the boundary conditions are applied at the interfaces ( $r = a$  ;  $z = \pm H/2$ ) which permits us to write the next relations.

$$\frac{J_0(k_r a)}{J_1(k_r a)} = - \frac{k_a \epsilon_r}{k_r \epsilon_a} \cdot \frac{K_0(k_a b) - \left\{ \frac{K_0(k_a b)}{I_0(k_a b)} \right\} \cdot I_0(k_a a)}{K_1(k_a b) + \left\{ \frac{K_0(k_a b)}{I_0(k_a b)} \right\} \cdot I_1(k_a a)} \quad (1)$$

$$\{ \beta \epsilon_a + \alpha_a \epsilon_r \cdot \operatorname{tg} \frac{\beta H}{2} \cdot \operatorname{th} \alpha_a (d_3 - H/2) \} \{ \alpha_s \epsilon_r \operatorname{th} \alpha_s (d_1 - H/2) - \beta \epsilon_s \operatorname{tg} \frac{\beta H}{2} \} + \{ \beta \epsilon_s + \alpha_s \epsilon_r \operatorname{tg} \frac{\beta H}{2} \cdot \operatorname{th} \alpha_s (d_1 - H/2) \} \{ \alpha_a \epsilon_r \operatorname{th} \alpha_a (d_3 - H/2) - \beta \epsilon_a \operatorname{tg} \frac{\beta H}{2} \} = 0$$

Where  $J_n$ ,  $K_n$  and  $I_n$  Bessel' functions and the propagation constants are related each other by the relations :

$$K_R^2 = \left(\frac{\omega}{C}\right)^2 \epsilon_R - \beta^2 = \alpha_a^2 + \left(\frac{\omega}{C}\right)^2 \epsilon_a = \alpha_s^2 + \left(\frac{\omega}{C}\right)^2 \epsilon_s \quad (2)$$

$$K_a^2 = \beta^2 - \left(\frac{\omega}{C}\right)^2 \epsilon_a$$

## 2. Quality factor $Q_0$

Calculating the stored energy in the different medium we can write :

$$Q_0^{-1} = Q_R^{-1} + Q_S^{-1} \cdot \frac{W_s}{W} + Q_m^{-1} \quad (Q_m = \frac{\bar{W}}{P_m}) \quad (3)$$

where  $(\bar{W}_s/\bar{W})$  is the ratio of the stored energy in the substrate and that stored in all the structure  $Q_R^{-1}$  and  $Q_S^{-1}$  represent the loss tangent of the materials (resonator and substrate). The loss tangents can be measured directly ; so to determine the Q factor, it's only necessary to calculate the stored energy and the metallic losses. These two quantities are very simple to evaluate when the electromagnetic fields are known.

The comparison, between the  $TE_{0\gamma\delta}$  and  $TM_{0\gamma\delta}$  mode frequency and quality factor is given on the figure (2).

## COUPLING

### 1. Coupling between a resonator and a microstrip line

The resonator excited in  $TM_{0\gamma\delta}$  mode is quite similar to an electrical dipole (figure 3).

The coupling can be of a magnetic type, so in the low frequency equivalent circuit, we place an inductance to describe the coupling (see figure 4).

The coupling factor, characterized by a load  $Q_L$  factor, is evaluated by determining the input impedance  $Z$  of the equivalent circuit, so we find

the next relation giving the  $Q_L$  factor as a function of the stored energy, the  $Q_0$  factor and the longitudinal component of the electric fields ( $E_z$ ):

$$Q_L = \frac{Q_0}{1 + \frac{Q_0}{4Z_C} \cdot \frac{|\int_{\text{resonator height}} E_z dz|}{\omega_0 \bar{W}}} \quad (4)$$

$Z_C$  is the characteristic impedance of the microstrip line.

The good accuracy between theoretical and experimental results is shown on the figure (5). We must precise that we have used the following approximation :

- no influence of the lateral metallic walls
- no perturbation of the electromagnetic fields by the microstrip line.

A very important factor, in this kind of coupling, is the axial unwedge between the resonator and the line (An unwedge equal to 1 mm involves a variation of  $Q_L$  factor equal to 3 and 5 % of this value).

### 2. Coupling between two dielectric resonators in a cut-off waveguide

For the design of bandpass filters using dielectric resonators, it's necessary to know the

coupling coefficient between two resonators. The figure (6) shows two resonators located inside a cut-off rectangular waveguide.

In this structure, we can consider two couplings : the first is the coupling by the resonator's fields and the second by the evanescent fields excited in the waveguide by the resonators. At low frequency, we have represented the structure by two resonant circuits (supposed without losses) coupled by a capacitance  $C$  (see fig.7).

If we consider two identical resonators we can obtain a simple relation for the coupling factor

. The coupling due to the resonators' fields, noted  $K_R$ , is obtained by using a similar concept developed by SKALICKY {1}. So after calculations, we obtain the next relations :

$$|K_R| = \frac{\epsilon_0(\epsilon_r - 1)}{2W} \iiint_V \vec{E}_{z1} \cdot \vec{E}_{z2} dv \quad (5)$$

where  $\vec{E}_{z1}$  and  $\vec{E}_{z2}$  represent, respectively, the longitudinal components of the electric fields outside the first resonator and inside the second resonator.

. The coupling due to the evanescent fields excited in the waveguide ( $K_G$ ) can be calculated as a function of the stored energy  $W$  and of the resonator electrical dipolar moment  $\vec{P}$ .

$$|K_G| = \frac{\vec{E}_2 \cdot \vec{P}_1}{2W} \quad (6)$$

where  $\vec{E}_2$  is the electric field created inside the second resonator by the first (this field is expressed as an infinite sum of waveguide modes {2})

The electrical dipolar moment  $\vec{P}_1$  is expressed as a function of the electrical fields inside the first resonator.

The theoretical results show that the  $K_G$  is very low compared with the  $K_R$  coefficient (the coupling with the free space is very difficult : if we calculate the waveguide impedance and the radiation impedance of an electrical dipole we note a great mismatching between them.

The figure (8) shows the good accuracy between the theory and the measurements. We must precise that we have considered the unperturbed fields of the resonators while the influence of the permittivity discontinuity is very important.

### 3. External $Q_e$ factor of a dielectric resonator inside a cut-off waveguide

A cylindrical dielectric resonator of diameter  $D$  and height  $H$  is placed inside an evanescent

rectangular waveguide A of width  $a$  and height  $b$ . One of the extremities is connected to one propagation waveguide B of the same height but of different width  $a$ .

When the propagating waveguide is excited, the  $(\vec{E}, \vec{H})$  fields in the waveguide will excite and induce a polarisation current density  $\vec{J}$  on the dielectric resonator. When the waveguide is excited in the  $TE_{10}$  mode, it can be seen that the resonator functions in the  $TM_{0\gamma\delta}$  mode.

The external  $Q_e$  factor characterizes the coupling between the waveguide B and the evanescent waveguide A loaded by a dielectric resonator. To determine this quantity, we develop the fields in the waveguide as an infinite sum of waveguide modes and write the radiated power :

$$\overline{P}_r = \frac{1}{2} \iint_S (\vec{E}_r \wedge \vec{H}_r^*) \cdot \vec{a}_z ds \quad (7)$$

$\vec{E}_r, \vec{H}_r$  radiated EM of the  $TE_{10}$  rectangular waveguide mode.

On figure 9, we see the good accuracy between theoretical and experimental results, concerning  $Q_e$  factor.

### CONCLUSION

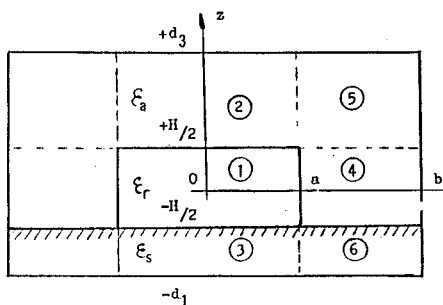
This paper shows that the  $TM_{0\gamma\delta}$  mode can be used as the  $TE_{0\gamma\delta}$  mode. The same structures of coupling have been analytically demonstrated and the study is in good agreement with experimentation.

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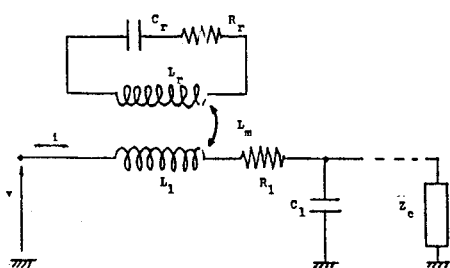
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### ACKNOWLEDGMENT

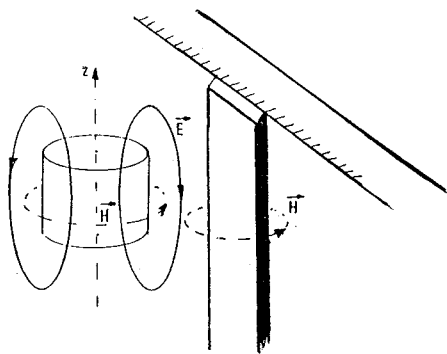
This work was supported by the National Telecommunications Study Center of Lannion C.N.E.T..



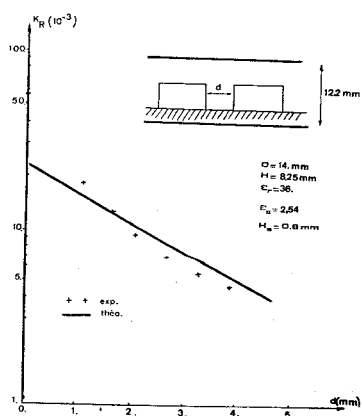
-Figure 1-



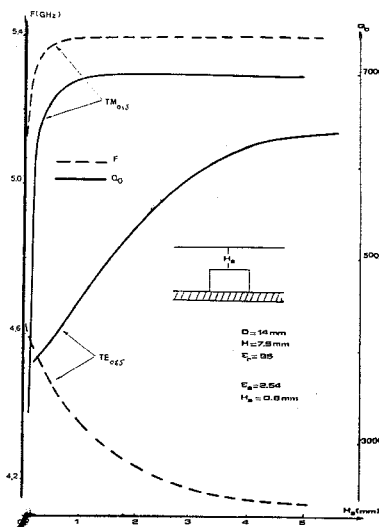
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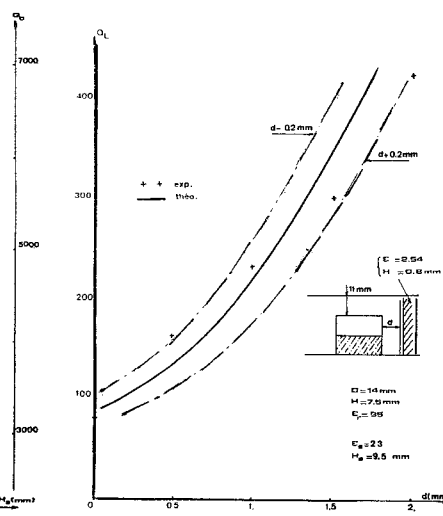
-Figure 3-



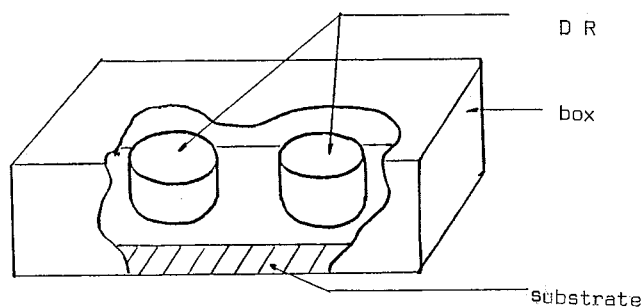
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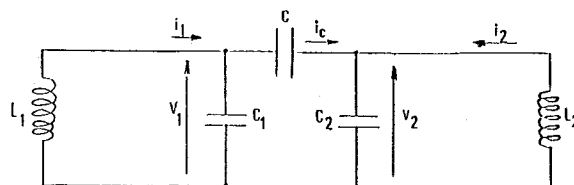
-Figure 2-



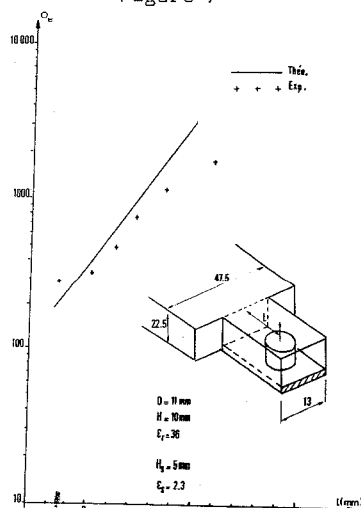
-Figure 5-



-Figure 6-



-Figure 7-



-Figure 9-